Circularly Polarized Microwave Cavity Filters*

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Summary—A new group of circularly polarized microwave cavity filters is described. With a single circularly polarized cavity, a reflectionless filter is achieved that couples nearly 100 per cent of the energy from the main waveguide at the cavity resonant frequency. Two degenerate cavity modes may be excited, to produce a circularly polarized field, by coupling to the transverse and longitudinal waveguide magnetic fields or to the transverse electric and magnetic waveguide fields.

A theoretical analysis is presented as well as experimental results. The loss between the band-pass terminals of the reflectionless cirlarly polarized filter is identical with the loss in a conventional reflection-type band-pass filter with the same bandwidth and cavity-wall losses. The null at resonance between the band-elimination terminals of the reflectionless circularly polarized filter is limited only by the asymmetries of the cavity and not by the cavity-wall losses. Design equations and curves are given for eight of the lower order, circularly cylindrical, degenerate cavity modes that are coupled to a rectangular waveguide at the point of circularly polarized waveguide magnetic fields.

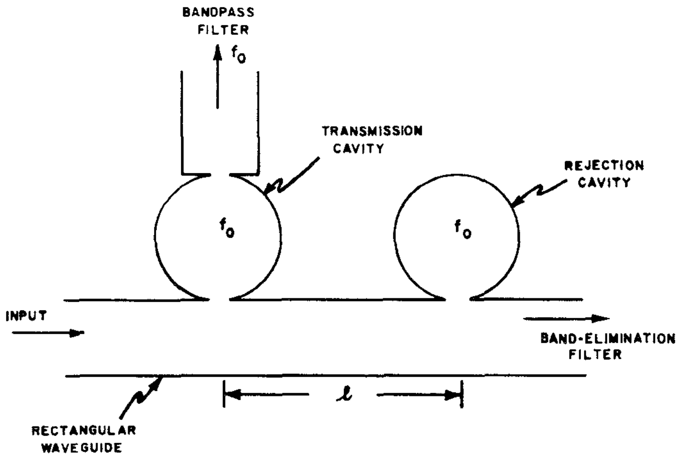

Fig. 1—Conventional waveguide diplexer.

I. Introduction

In the conventional design of three terminal microwave cavity filters, at least two cavities are required to produce a matched filter capable of coupling nearly 100 per cent of the power from the main waveguide at resonance. Fig. 1 illustrates this method of diplexing.

A low-loss "rejection" cavity must be spaced a critical distance from the main cavity. The single circularly polarized cavity described in this paper can replace these two cavities and perform the same function. The theoretical loss for the two systems is the same (assuming that each system has the same band-width and cavity loss). However, the isolation at resonance for the conventional two-cavity diplexer is limited by the rejection cavity losses, while the isolation of the circularly polarized cavity is limited only by asymmetries of the cavity.

There are many applications for the circularly polarized single-cavity filter. Since this filter is essentially reflectionless, several may be placed in series without producing large reflections. The circularly polarized cavity may be used as a passive duplexer for a circularly polarized radar as well as a nonreciprocal, tunable filter by placing a ferrite in the circularly polarized magnetic fields in the cavity. The use of a ferrite also permits the design of a narrow-band duplexer for use with linearly polarized radar systems.

II. Principle of Operation

One configuration of the circularly polarized microwave cavity filter is shown in Fig. 2(a). It has four waveguide ports and only one circularly cylindrical cavity.[1] The cavity is designed to operate with two orthogonal, degenerate, TE_{112} modes (several other cavity modes will work just as well). Two circular coupling holes are centered at each end of the cavity, but they are offset from the waveguide centerline. Fig. 2(b) shows the cavity rf magnetic fields;[2] these fields are coupled to the transverse (H_x) and longitudinal (H_z) waveguide magnetic fields. In this example, the circular coupling holes are placed in the waveguides at the point of circular polarization of the waveguide magnetic fields [Fig. 2(c)].

The operation of the filter is as follows. Energy enters the waveguide at port A [Fig. 2(a)] and excites the cavity in a circularly polarized TE_{112} mode, since H_x and H_z in the waveguide are 90 degrees out of time phase. The excited cavity radiates into waveguides A-B and C-D. However, due to the direction of circulation of the cavity modes, the cavity will radiate only toward waveguide ports B and C. Therefore, with an input signal at port A, there should be no reflection back to port A, and the output energy will divide between ports B and C.

The fact that the filter is reflectionless is important, but an even more important characteristic is that nearly 100 per cent of the energy entering port A can be coupled into arm C at the cavity resonant frequency (neglecting cavity-wall losses). Normally, a single-mode cavity must share the input energy with the load in arm B, since the two are either in series or in parallel. However,

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[1] This type of filter is being investigated independently by S. B. Cohn and F. S. Coale at the Stanford Res. Inst. See "Directional channel-separation filters," 1956 IRE Convention Record, part 5, pp. 106–112.

[2] S. Ramo and J. Whinnery, "Fields and Waves in Modern Radio," John Wiley and Sons, Inc., New York, N. Y.; 1947.

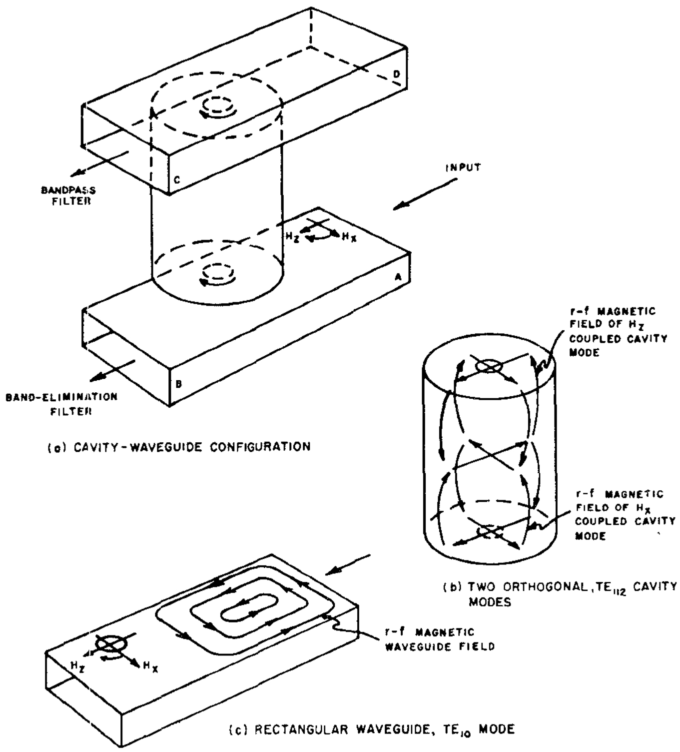

Fig. 2—A circularly polarized microwave cavity filter using magnetic field coupling.

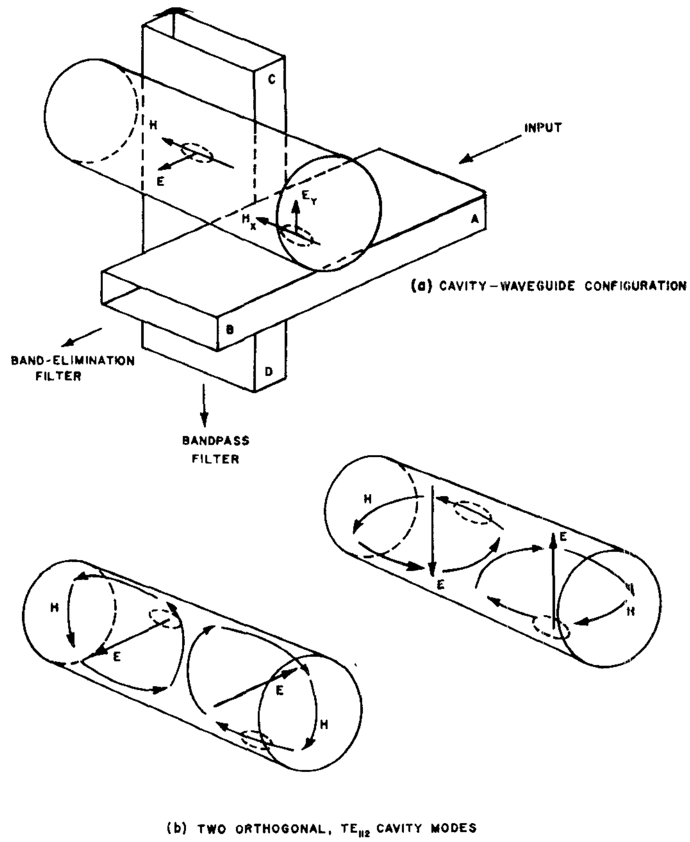

Fig. 3—A circularly polarized microwave cavity filter using electric and magnetic field coupling.

with the dual-mode cavity, each cavity mode couples out one half of the incident energy at resonance, and this produces a null in arm B. Far off resonance, the cavity will not couple to the waveguide, and all the energy entering port A will leave port B. The result is a single-cavity, reflectionless filter that is band-elimination between ports A-B and band-pass between ports A-C.

In the preceding example, a circularly polarized cavity mode is attained by coupling to the circularly polarized waveguide magnetic fields. Another arrangement that will produce a circularly polarized cavity mode is shown in Fig. 3. The cavity is designed to operate with two orthogonal, degenerate, TE_{112} modes. Two coupling holes are spaced between the end plates of the cavity and on the waveguide centerline. Fig. 3(b) shows the cavity rf magnetic and electric fields. These fields are coupled to the waveguide transverse magnetic (H_x) and transverse electric (E_y) fields. In the waveguide, the transverse components of electric and magnetic fields are in phase, but in a cavity, E and H for one mode are 90 degrees out of time phase. Therefore, the magnetic field of the E_y-coupled cavity mode lags (or leads, depending upon assigned directions of mode orientation) the magnetic field of the H_x-coupled cavity mode by 90 degrees. The coupling holes are centered in the waveguide, to couple to the peak of the transverse waveguide fields. The slot is adjusted in shape to produce equal coupling to the two cavity modes, because the electric and magnetic fields are different in the waveguide as well as in the cavity.

The operation of the E_y- and H_x-coupled cavity is similar to the H_x- and H_z-coupled cavity. Energy entering port A excites the cavity in a circularly polarized TE_{112} mode. The excited cavity then radiates into waveguides A-B and C-D. However, the radiations of the two cavity modes back to port A will cancel, since they are 180 degrees out of time phase. In waveguide C-D, the energy from both cavity modes cancels at port C, but adds at port D. As before, a reflectionless band-elimination and band-pass filter results.

Any combination of the above two coupling methods may be used, as well as different waveguide orientations. The coupling holes in the E and H coupling technique do not have to be at right angles but may be opposite each other, provided that the cavity mode configuration is favorable. Also many degenerate cavity modes have more than one position on the end plate at which a circularly polarized magnetic field results.

III. Analysis of Dual-Mode Cavity With Four Waveguide Ports

Fig. 2 is an example of a cavity system to which the following analysis applies. Several other cavity modes, coupling-hole shapes, positions, and waveguide orientations will work just as well. Some of these will be described later. The important requirement of the reflectionless filter is that two orthogonal waveguide fields are coupled to two orthogonal cavity modes that are resonant at the same frequency and are 90 degrees out of time phase. The coupling to the two cavity modes must take place in the same transverse waveguide plane. As the analysis will show, the coupling-hole position or shape must be adjusted, to produce an equal coupling to each cavity mode. In general, the analysis also applies to the cavity system shown in Fig. 3; however, the equivalent circuit will be slightly different. Although this analysis applies primarily to the dual-mode cavity with four waveguide ports and two coupling holes, only slight modifications are required, to cover the case of a cavity system with two waveguide ports and only one coupling hole.

The conventional microwave cavity filter analysis will be reviewed briefly, since most of these circuit equations and equivalent circuits can be applied to the solution of the reflectionless dual-mode cavity filter.

A. Series Coupling

A cavity, coupled only to the transverse magnetic field (H_x) of the TE_{10} rectangular waveguide mode, acts in series with the waveguide. It is assumed that all cavity modes except the desired one are far off resonance and will not affect the equivalent circuit. The waveguide is represented as a transmission line of characteristic impedance Z_0 and the cavity (H-coupled) presents an impedance Z_c in series with the transmission line. Fig. 4(a) illustrates the cavity-waveguide arrangement. Either the impedance Z_c or the normalized impedance z is a parallel tuned circuit as shown in Fig. 4(b). However, a more useful equivalent circuit [Fig. 4(c)] is obtained by letting the cavity internal impedance be a series tuned circuit and then having a special transformer reverse this impedance, so that the transmission line (TL) sees a parallel tuned circuit.[3,4] The normalized impedance

$$\frac{1}{Q_c} + j\left(\frac{f}{f_0} - \frac{f_0}{f}\right)$$

is the "internal" cavity loss and reactance. The cavity Q, Q_c, represents the cavity losses, such as a lossy cavity dielectric and cavity wall losses. The cavity current I_c represents the actual current in the cavity walls and, hence, is proportional to the magnitude of the cavity fields. The relations between the cavity and TL normalized currents and voltages are

$$\left.\begin{aligned} I_c &= \pm j\sqrt{Q_w}V_2 \\ V_c &= \pm j\frac{I_1}{\sqrt{Q_w}} \\ \frac{V_c}{I_c} &= \frac{1}{Q_c} + j\left(\frac{f}{f_0} - \frac{f_0}{f}\right) \end{aligned}\right\} \qquad (1)$$

[3] H. A. Bethe, "Theory of Side Windows in Waveguides," M.I.T. Rad. Lab. Rep. No. 43-27.

[4] H. A. Bethe, "Excitation of Cavities Through Windows," M.I.T. Rad. Lab. Rep. No. 43-30.

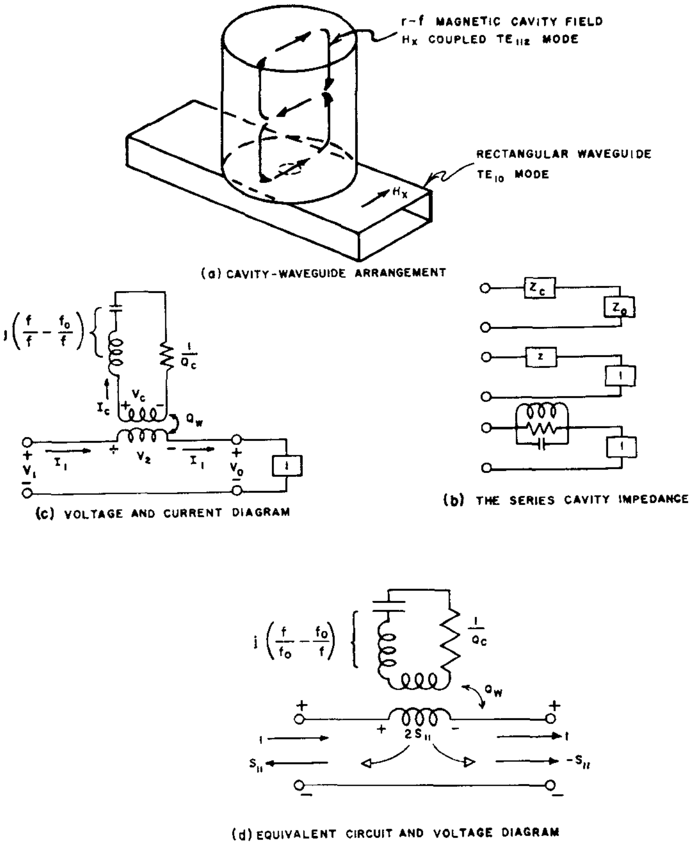

Fig. 4—Series-coupled cavity with two waveguide ports.

where

Q_w = window Q or window-coupling factor[5]
Q_c = cavity Q
f_0 = cavity resonant frequency
f = frequency.

The signs of the internal cavity current, I_c, and voltage, V_c, depend upon the assigned polarity of the cavity mode. The normalized cavity impedance in series with the TL is

$$z = \frac{V_2}{I_1} = \frac{1}{Q_w}\frac{I_c}{V_c} = \frac{1}{\dfrac{Q_w}{Q_c} + jQ_w\left(\dfrac{f}{f_0} - \dfrac{f_0}{f}\right)}. \qquad (2)$$

The special transformer, represented by the window coupling factor Q_w, has no impedance itself, but only inverts the "internal" cavity impedance. The reciprocal of the window-coupling factor Q_w represents the amount of coupling between the cavity and the waveguide. Without a coupling hole, the coupling is zero, Q_w is infinite, and the cavity impedance z is zero. As the coupling hole is made larger Q_w decreases and z increases. H. A. Bethe has determined Q_w, and numerical values of Q_w are given in Section V. It should be noted that in (2) the value of Q_w differs by a factor of two from that

[5] Q_w is the same as the external or radiation Q given by C. G. Montgomery, "Technique of Microwave Measurements," McGraw-Hill Book Co., Inc., New York, N. Y.; 1947.

for Q_w in Bethe's article. This is because of the definition of Q_w given in (2).

For an impedance (or cavity) in series with the TL the normalized input impedance z_i with a matched load is $z+1$. The voltage scattering matrix coefficients are

$$S_{11} = \frac{z_i - 1}{z_i + 1} = \frac{z}{2 + z}, \tag{3}$$

and

$$S_{12} = 1 - S_{11}.$$

If a unit voltage is incident upon the cavity system, it will travel along the waveguide, excite the cavity, and proceed to the output terminal. However, the excited cavity radiates back into the waveguide with the voltage equally divided between input and output terminals. The voltage radiated or reflected back to the input terminals is $+S_{11}$, and the net voltage at the input terminals is $1+S_{11}$. The voltage appearing at the output terminals is $1-S_{11}$. The voltage $-S_{11}$ is due to the cavity radiation or cavity impedance [see Fig. 4(d)], The concept of the cavity radiating back into the waveguide to produce two traveling voltages, S_{11} and $-S_{11}$, is important and is required when the series and parallel coupled cavities are combined. Fig. 5 illustrates a more complicated series-coupled cavity system; it also represents the series-coupling equivalent circuit for the dual-mode cavity system. With a unit incident voltage at port A, the cavity will be excited, and the incident voltage will appear at port B. The excited cavity will radiate into waveguides A-B and C-D. The resulting voltages, due to the cavity, will be S_{11}, $-S_{11}$, S_{13}, and $-S_{13}$. The normalized cavity impedance in series with waveguide A-B is

$$z = \frac{1}{\dfrac{Q_{w1}}{Q_{c1}} + jQ_{w1}\left(\dfrac{f}{f_1} - \dfrac{f_1}{f}\right) + \dfrac{1}{2}\dfrac{Q_{w1}}{Q_{w2}}}. \tag{4}$$

There is a factor of two in (4), since the cavity coupled to waveguide C-D sees ports C and D in series. The coefficient S_{11} is

$$S_{11} = \frac{1}{1 + \dfrac{Q_{w1}}{Q_{w2}} + \dfrac{2Q_{w1}}{Q_{c1}} + j2Q_{w1}\left(\dfrac{f}{f_1} - \dfrac{f_1}{f}\right)}. \tag{5}$$

The coefficient S_{13} is obtained as follows. The voltage across the cavity terminals in waveguide A-B is $2S_{11}$, and the voltage across the cavity terminals in waveguide C-D is $2S_{13}$. The "interval" cavity current I_c is

$$I_c = j2S_{11}\sqrt{Q_{w1}} = j2S_{13}\sqrt{Q_{w2}},$$

therefore,

$$S_{13} = \sqrt{\frac{Q_{w1}}{Q_{w2}}}\, S_{11}. \tag{6}$$

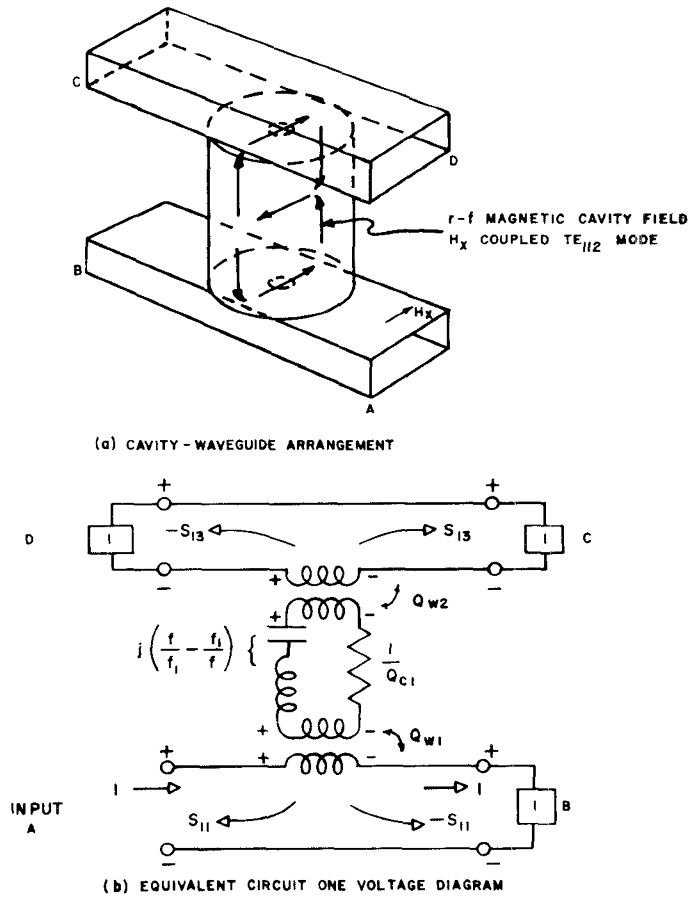

Fig. 5—Series-coupled cavity with four waveguide ports.

The signs of the voltage S_{13} and cavity current I_c may be plus or minus, depending upon the direction of the magnetic fields in the cavity. The sign of S_{13} is chosen as plus, and the same sign convention is used in the parallel-coupled cavity.

B. Parallel Coupling

Let us consider the cavity shown in Fig. 6(a), which couples only longitudinal magnetic fields (H_z) of two waveguides. In the equivalent circuit [Fig. 6(b)] the cavity is in parallel with the TL. This is because a short circuit in arm B, placed one half of a guide wavelength from the cavity coupling hole, would produce a standing wave with zero H_z (and E_y) at the coupling hole. The cavity would not be excited—hence, it would be shorted out as if it were in parallel with the TL. A quarter wavelength TL is needed in the equivalent circuit, to invert the impedance and to present a 90-degree time lag in the voltage reaching the cavity (the H_z waveguide component is 90 degrees behind the H_x and E_y components, which are assigned zero time phase). The internal cavity current for this type of coupling lags by 90 degrees the cavity current for the series-coupled cavity.

With a unit incident voltage at port A, shown in Fig. 6(b), the cavity causes voltages T_{11}, T_{11}, T_{13}, and T_{13} to appear at the four waveguide ports. Normalized, parallel cavity admittance seen by waveguide A-B is

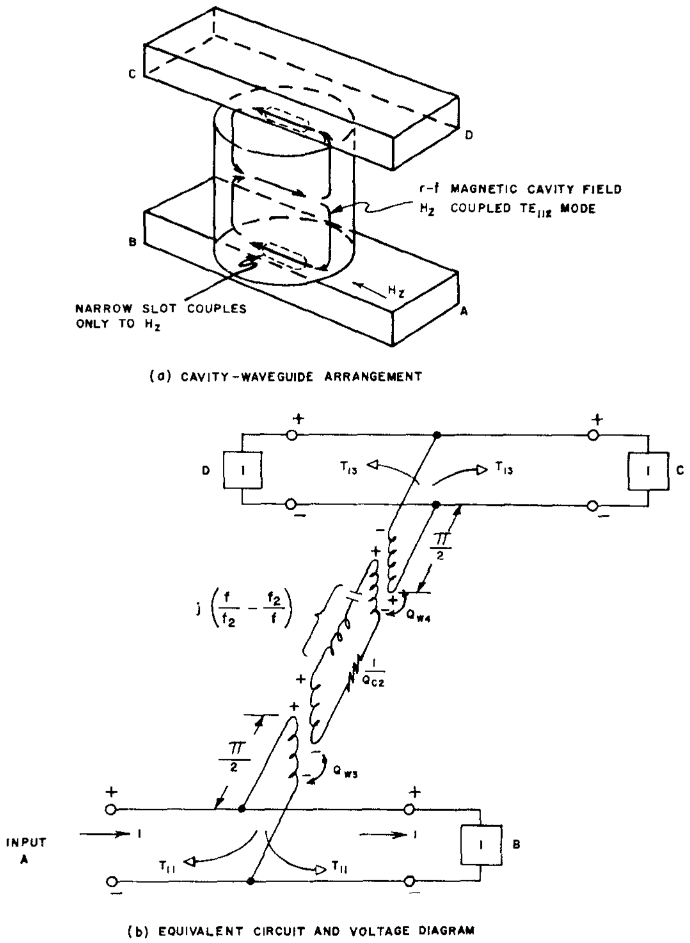

Fig. 6—Parallel-coupled cavity with four waveguide ports.

$$y = \frac{1}{\frac{Q_{w3}}{Q_{c2}} + jQ_{w3}\left(\frac{f}{f_2} - \frac{f_2}{f}\right) + \frac{1}{2}\frac{Q_{w3}}{Q_{w4}}}. \tag{7}$$

The coefficient T_{11} is

$$T_{11} = \frac{1-(y+1)}{1+(y+1)} = -\frac{y}{2+y} \tag{8}$$

$$T_{11} = \frac{-1}{1 + \frac{Q_{w3}}{Q_{w4}} + \frac{2Q_{w3}}{Q_{c2}} + j2Q_{w3}\left(\frac{f}{f_2} - \frac{f_2}{f}\right)}. \tag{9}$$

As before (with considerable manipulation),

$$T_{13} = -\sqrt{\frac{Q_{w3}}{Q_{w4}}}\,T_{11}. \tag{10}$$

A negative sign was added to (10), to make it consistent with the assumed polarity of S_{13}.

C. *Circularly Polarized Coupling*

The complex cavity shown in Fig. 2 can now be analyzed. It should be noted that the coupling hole is offset from the waveguide centerline and couples the orthogonal waveguide H_x and H_z components to the two orthogonal, degenerate, TE_{112} circularly cylindrical cavity modes. There is a component of electric field (E_y) in the coupling hole, due to the waveguide fields, but, since the TE_{112} cavity mode has a zero electric field at the center of the end plate, this component does not provide additional coupling to this mode, and the other cavity modes are far off resonance.

The transverse waveguide field H_x couples to the first cavity mode (with constants Q_{w1}, Q_{w2}, Q_{c1}, and f_1). The longitudinal waveguide field H_z couples to the second cavity mode (with constants Q_{w3}, Q_{w4}, Q_{c2}, and f_2). There is no interaction between the series- and parallel-coupled cavity modes, because the cavity modes are orthogonal and are excited by orthogonal fields in the waveguide. Furthermore, the waveguide ports B, C, and D are terminated in matched loads. With a unit incident voltage at port A, the series cavity produces voltages S_{11}, $-S_{11}$, S_{13}, and $-S_{13}$, and the parallel cavity produces voltages T_{11}, T_{11}, T_{13}, and T_{13}. The resulting voltages at each of the waveguide ports are listed below.

Port	Voltage
A	Incident voltage $=1$
A	Reflected voltage $V_a = S_{11} + T_{11}$
B	Output voltage $V_b = 1 - S_{11} + T_{11}$
C	Output voltage $V_c = S_{13} + T_{13}$
D	Output voltage $V_d = -S_{13} + T_{13}$

A comparison of (5) and (8) shows that, if the normalized, series cavity impedance is equal to the normalized, parallel cavity admittance, then

$$z = y$$

and

$$S_{11} = -T_{11}$$
$$S_{13} = +T_{13}$$

the resulting output voltages (for a unit incident voltage at port A) are

$$V_a = 0$$
$$V_b = 1 - 2S_{11} = \frac{2-z}{2+z}$$
$$V_c = 2S_{13} = \frac{2z}{2+z}\sqrt{\frac{Q_{w1}}{Q_{w2}}}$$
$$V_d = 0. \tag{11}$$

The requirements for a reflectionless filter are

$$Q_{w1} = Q_{w3} \text{ (in waveguide A-B)}$$
$$Q_{w2} = Q_{w4} \text{ (in waveguide C-D)}$$
$$Q_{c1} = Q_{c2}$$
$$f_1 = f_2. \tag{12}$$

Therefore, a reflectionless filter results when each cavity mode has the same resonant frequency, loss, and coupling to the waveguide fields. The electromagnetic fields in the two cavity modes are equal in amplitude, but are

90 degrees out of phase; hence, a true circularly polarized mode results. If a circular coupling hole is used, it must be placed at a position of circular polarization of waveguide magnetic fields, since the cavity modes are identical. If an asymmetrical slot is used, it must be placed in the waveguide in a manner that will compensate for its symmetry and produce equal coupling.

The power transfer characteristics for the circularly polarized filter can be expressed in simplified terms. Assume a unit incident power (of frequency f) at port A; then, the resulting output powers are

$$P_a = 0$$
$$P_b = \frac{m + x^2}{1 + x^2}$$
$$P_c = \frac{n}{1 + x^2}$$
$$P_d = 0, \tag{13}$$

where

$$P = |V|^2 = \frac{\text{Output Power}}{\text{Incident Power at A}}$$

$$x = \frac{2(f - f_0)}{BW} \tag{14}$$

$$m = \left[\frac{\frac{1}{Q_{w2}} - \frac{1}{Q_{w1}} + \frac{2}{Q_c}}{\frac{1}{Q_{w2}} + \frac{1}{Q_{w1}} + \frac{2}{Q_c}}\right]^2 \sim 0 \tag{15}$$

$$n = \frac{1}{\left[\frac{1}{2}\left(\sqrt{\frac{Q_{w1}}{Q_{w2}}} + \sqrt{\frac{Q_{w2}}{Q_{w1}}}\right) + \frac{\sqrt{Q_{w1}Q_{w2}}}{Q_c}\right]^2} \sim 1 \tag{16}$$

$$\frac{1}{Q_L} = \frac{BW}{f_0} = \frac{1}{2}\left(\frac{1}{Q_{w1}} + \frac{1}{Q_{w2}}\right) + \frac{1}{Q_c} \tag{17}$$

BW = 3-db bandwidth.

Eq. (13) predicts three important characteristics of this system:

1) Throughout the operating frequency band including resonance there will be practically no reflection back to port A.
2) At resonance there will be a power null of amplitude m at port B. This results in a "reflectionless band-elimination filter."
3) At resonance, energy will be coupled through the cavity and will produce power output of amplitude n at port C. This results in a "reflectionless band-pass filter."

D. Equivalent Circuit

An equivalent circuit for the filter illustrated in Fig. 2 is shown in Fig. 7. The special transformer for the series-coupled cavity is center-tapped, and each leg has an equivalent window-coupling factor of $4Q_w$, because the coupling is actually proportional to the reciprocal of the square root of Q_w. Different currents are in each leg of the series-coupling transformer, and this produces the effect required to achieve unity coupling between ports A and C (neglecting losses). By observing the electromagnetic fields and the type of coupling in any cavity and waveguide system, the proper equivalent circuit can be determined as well as the direction of power flow at resonance.

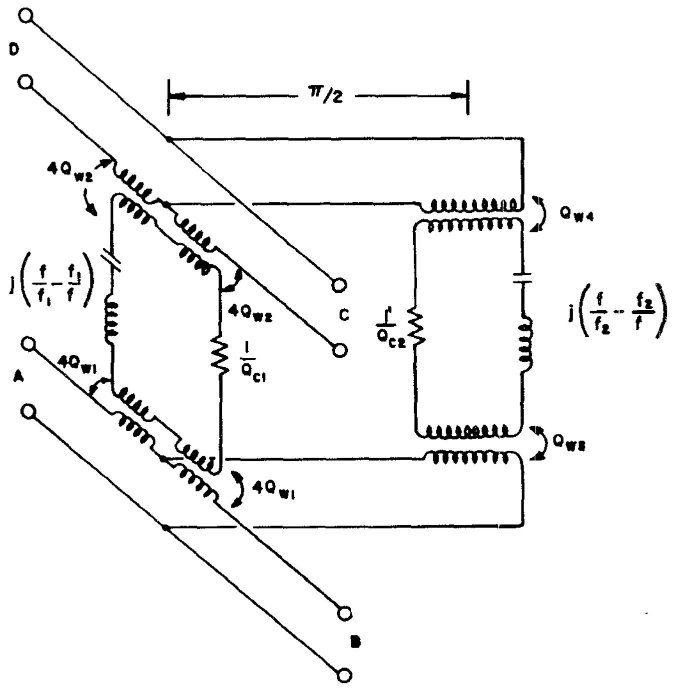

Fig. 7—Equivalent circuit for the dual-mode cavity with four ports.

E. Optimum Band-pass Filter

The condition of maximum power transfer between ports A and C is achieved when coupling holes of the same size are placed in waveguides A-B and C-D. The resulting equations are

$$Q_w = Q_{w1} = Q_{w2} = Q_{w3} = Q_{w4}$$
$$\frac{1}{Q_L} = \frac{1}{Q_w} + \frac{1}{Q_c} = \frac{BW}{f_0}$$
$$n = \left[1 - \frac{Q_L}{Q_c}\right]^2$$
$$m = \left[\frac{Q_L}{Q_c}\right]^2. \tag{18}$$

With zero cavity-wall losses, unity coupling from the main waveguide at resonance can theoretically be achieved. The system is symmetrical, inasmuch as the operations from A to C, C to A, B to D, and D to B are identical. It should also be pointed out that the band-pass filter loss for the circularly polarized filter is identical with the loss in a conventional band-pass filter or with the two cavity filters shown in Fig. 1 (assuming the same bandwidth and cavity Q). Eq. (18) also indicates that, with maximum power transfer between ports A and C (Fig. 2), the isolation at resonance between ports A and B is not optimum. Isolation at resonance can theoretically be made to approach infinity with an unnoticeable increase in loss between ports A and C.

F. Optimum Band-Elimination Filter

The isolation between ports A and B can theoretically be made infinite by making the coupling hole in waveguide C-D slightly smaller than that in waveguide A-B. The resulting equations are

$$Q_{w1} = Q_{w3} = Q_L$$

$$Q_{w2} = Q_{w4} = \frac{Q_{w1}}{1 - \dfrac{2Q_{w1}}{Q_c}} \geq Q_{w1}$$

$$n = 1 - \frac{2Q_L}{Q_c}$$

$$m = 0. \tag{19}$$

In a conventional band-elimination filter, the cavity loss limits the isolation at resonance. In the circularly polarized filter, there is no theoretical limitation on isolation, only a practical limitation. When the isolation is larger than 30 or 40 db, the null is usually too sharp to be of any use. With coupling holes of different sizes, there is optimum isolation only between ports A and B or B and A.

G. Two-Port Band-Elimination Filter

The equations for the band-elimination filter, when waveguide C-D is removed, are

$$Q_{w2} = Q_{w4} = \infty$$

$$\frac{1}{Q_L} = \frac{1}{2Q_{w1}} + \frac{1}{Q_c}$$

$$\sqrt{m} = \frac{\dfrac{Q_{w1}}{Q_c} - \dfrac{1}{2}}{\dfrac{Q_{w1}}{Q_c} + \dfrac{1}{2}}. \tag{20}$$

The null in output power at resonance can be made zero by adding a lossy material to the cavity. If

$$Q_c = 2Q_{w1}, \tag{21}$$

then

$$Q_L = Q_{w1}$$

$$m = 0$$

$$P_b = \frac{x^2}{1 + x^2}.$$

Now the cavity Q, Q_c, includes the lossy material in the cavity. If too much loss is added to the cavity, the null will not be zero. Only a specified cavity loss will produce a zero null, as indicated by (21).

If an isolation of 30 db is required over a narrow frequency band, the 3-db bandwidth will have to be large, because 30 db is maintained only over 3.16 per cent of the 3-db bandwidth ($m=0$). If two identical cavity systems having a zero null and the same resonant frequency are placed in series along a waveguide (the spacing is not critical, since each cavity system is reflectionless), the normalized output power is

$$P_b = \left[\frac{x^2}{1 + x^2}\right]^2 = \left[\frac{u^2}{0.4142 + u^2}\right]^2 \tag{22}$$

where

$$x = \frac{2(f - f_0)}{f_0} Q_{w1}$$

$$u = \frac{2(f - f_0)}{f_0} Q_L$$

$$Q_L = \sqrt{\sqrt{2} - 1}\, Q_{w1}$$

$$Q_c = 2Q_{w1}.$$

Therefore, an isolation of 30 db is maintained over 11.6 per cent of the 3-db bandwidth. If the resonant frequencies of the two cavity systems are separated until the response has a 30-db ripple, then the ratio of the 30-db bandwidth to the 3-db bandwidth increases to 16.4 per cent. Note that a single-cavity system has two cavity modes resonant at the same frequency.

If two cavity systems are placed in parallel (two coupling holes in the same transverse waveguide plane), then, with a ripple in the response, the theoretical ratio of the 30-db bandwidth to 3-db bandwidth is 24.8 per cent.

IV. Discussion of Cavity Modes and Coupling

In section III, it has been shown that, when a waveguide excites a circularly polarized cavity field by means of two orthogonal couplings in the same transverse waveguide plane, a reflectionless microwave filter results. Therefore, a single-cavity, reflectionless filter may be constructed using a spherical, a square, or a circular cylindrical cavity, with aperture, loop, or probe coupling. Loop coupling would be advantageous at the lower frequencies. However, the following discussion is restricted to aperture-coupled, circular cylindrical cavities.

A given cavity shape can support an infinite number of cavity modes. The use of higher order cavity modes will, in general, result in the following characteristics: 1) higher cavity Q, *i.e.*, lower wall losses; 2) larger size; 3) smaller available coupling (smaller bandwidth for a given coupling-hole size); and 4) other modes that have resonant frequencies close to that of the desired mode. Therefore, it seems expedient at this time to list the lower order cavity modes that may be circularly polarized. Eight of the lower order circular cylindrical cavity modes that, when circularly polarized, have a circularly polarized magnetic field at the center of the end plate

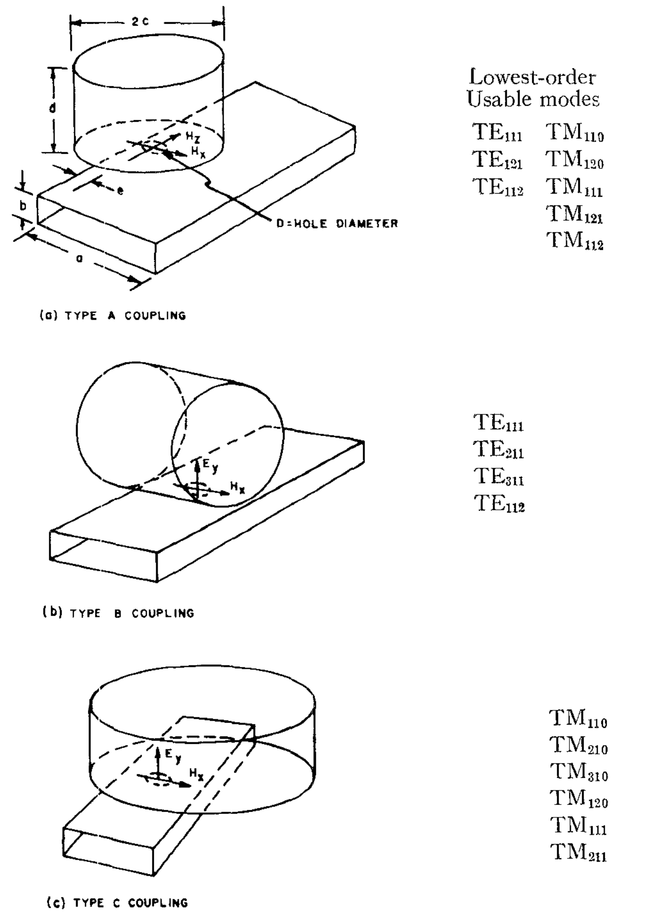

Fig. 8—Types of coupling for circularly polarized cavity modes using circularly cylindrical cavities.

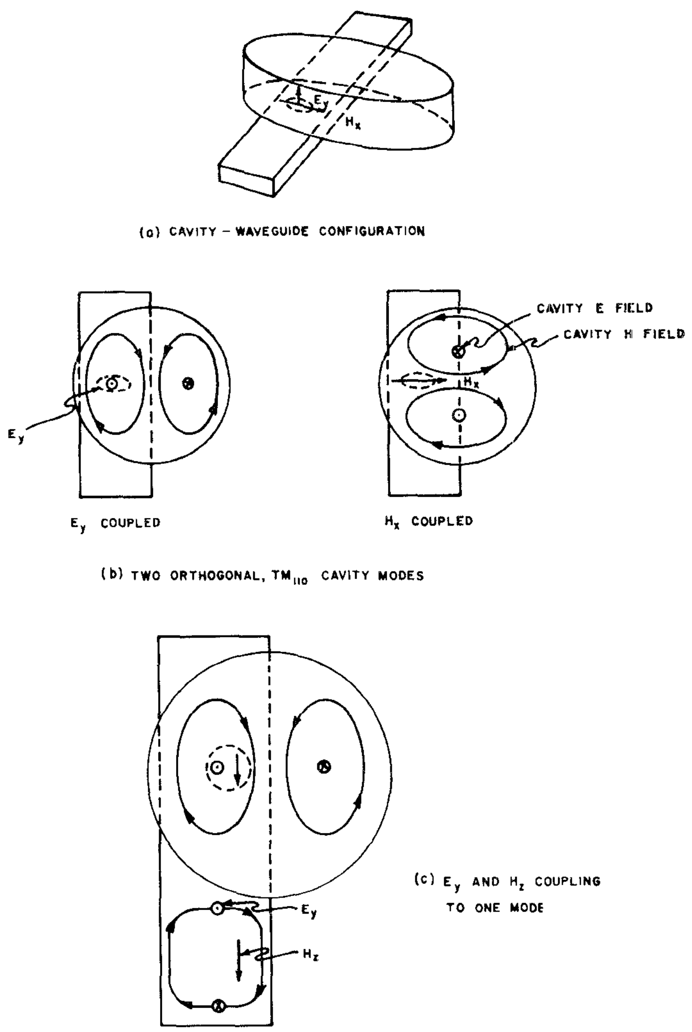

Fig. 9—Circularly polarized cavity, TM_{110} mode.

are listed in Fig. 8(a). These modes are coupled to the H_x and H_z waveguide fields (Type A coupling). Four of the lower order, degenerate, TE modes that may be E_y- and H_z-coupled (Type B coupling) are given in Fig. 8(b), and six of the lower order, degenerate, TM modes that may be E_y- and H_x-coupled (Type C coupling) are listed in Fig. 8(c). For the cavity-waveguide configuration illustrated in Fig. 8(b), only the TE cavity modes have a longitudinal magnetic field. Similarly in Fig. 8(c), only the TM cavity modes have a longitudinal electric field. Fig. 9(a) and 9(b) illustrates Type C coupling to two orthogonal, TM_{110} cavity modes. The coupling hole is on the waveguide centerline and at the position of maximum electric field in the cavity for one TM_{110} mode. The shape of the coupling hole is adjusted, to vary the electric and magnetic polarization of the aperture, and to obtain a circularly polarized cavity field. The position of the coupling hole does not permit H_z coupling between the waveguide and the E_y-coupled cavity mode. However, it may be advantageous to allow a certain amount of H_z coupling. For example, if it were desired to use a circular coupling hole in Fig. 9(a) for all ratios of cavity diameter to cavity length, then the position of the circular hole would be varied in the waveguide and cavity, to produce a circularly polarized cavity mode. In this case, a combination of H_x, E_y, and H_z coupling is being used. As illustrated in Fig. 9(c), a slight movement of the coupling hole in the waveguide and cavity allows a small amount of H_z coupling. The H_z coupling affects only the E_y-coupled mode, and it either adds or subtracts to the amount of E_y coupling. If the hole in the waveguide and cavity is moved in the same direction, as in Fig. 9(c), then the H_z coupling adds to the E_y coupling, since H_z lags E_y by 90 degrees in the waveguide, and H lags E by 90 degrees in the cavity for the given field directions.

The coupling hole in a cavity alters the electromagnetic cavity fields so that the cavity resonant frequency is different from the resonant frequency calculated from the cavity diameter and length. Bethe's analysis[4] shows that a cavity mode with a magnetic field in the aperture lowers the resonant frequency, and that an electric field in the coupling aperture raises the resonant frequency. Therefore, with Type A coupling, each cavity-mode resonant frequency will be lowered by nearly the same amount. For large coupling holes, the magnetic field of one cavity mode in the coupling aperture sees the waveguide wall in closer proximity than does the magnetic field of the other cavity mode. In such a case, a small tuning screw is required for one of the cavity modes, in order to adjust both modes to the same resonant frequency. With Types B and C coupling, the resonant frequency of the two cavity modes is changed in opposite directions, due to the coupling hole, and a larger tuning screw for one of the modes will be required.

However, when a symmetrical coupling system is used (as in Fig. 3), each cavity mode uses both E and H coupling, and the resonant frequency of each mode should be the same. Even though the resonant frequencies are theoretically the same, very small tuning screws are usually required to compensate for manufacturing tolerances.

V. Window-Coupling Factor, Q_w

The value of the window-coupling factor Q_w is a measure of the coupling between the waveguide and cavity. This coupling and the cavity losses determine the bandwidth of a particular filter, as was shown in section III. The basic equation for Q_w has been determined by Bethe using small-coupling-hole theory (the hole size is much smaller than the wavelength, and the hole is in a uniform cavity and waveguide field).[3,4,6] As the coupling hole increases in size, the error in the calculated value of Q_w increases. With this restriction in mind, theoretical curves of Q_w, for Type A coupling, are shown in Fig. 10. Theoretical curves of Q_w, for Types B and C coupling, are omitted, since their derivation is much more involved.

The values of Q_w for the cavity diameter-length ratios given in Fig. 10 were computed for a standard X-band waveguide (0.9 by 0.4 inch) with a cavity resonant frequency of 9000 mc. At this frequency, the point of circularly polarized magnetic field in the waveguide is 0.234 inch from one side wall. The maximum coupling-hole diameter is then 0.468 inch, which is the hole size used for the curves shown in Fig. 10. To find the window-coupling factor[7] for a hole with a diameter of D inches, multiply the value of Q_w in Fig. 10 by

$$\left[\frac{0.468}{D}\right]^6. \qquad (23)$$

These theoretical curves indicate the minimum value of Q_w attainable for a given cavity diameter-to-length ratio. For example, if a 20-mc bandwidth was required ($Q_L=450$) using a TE_{112} mode in a band-pass filter, the value of Q_w would be approximately 465 [from (18) and by assuming that $Q_c=14{,}000$]. Then (from Fig. 10), it is determined that the cavity would have to have a diameter-to-length ratio approximately between 0.4 and 1.4. If the cavity diameter-to-length ratio is 0.8, then the required coupling-hole diameter is

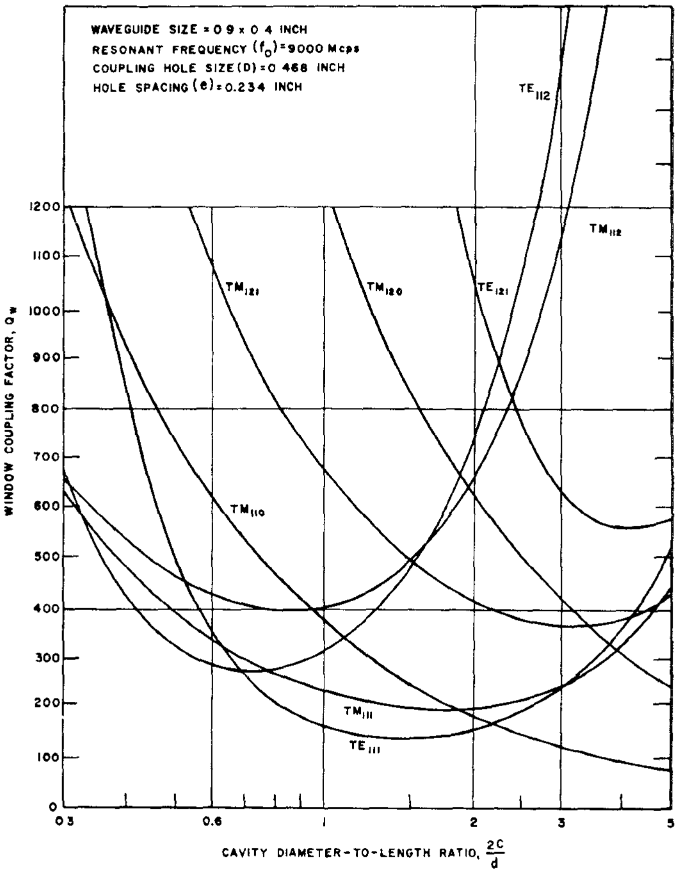

Fig. 10—Window coupling factors for circularly cylindrical cavities using type A coupling.

$$D = 0.468\left[\frac{280}{465}\right]^{1/6} = 0.430 \text{ inch.}$$

Actually, for a window size this large, the value of the curves for Q_w may be as much as 30 per cent in error. The window size should probably be made 0.440 inch instead of 0.430 inch. The optimum diameter would have to be determined experimentally. Therefore, Fig. 10 should be used only as a design aid. However, the curves are useful in determining approximate values of Q_w or D, as well as in determining which cavity modes do not have sufficient coupling, for a given bandwidth requirement. Theoretical curves for both cavity Q and the resonant frequency of most of the modes shown in Fig. 10 are given by Montgomery.[5]

Because of its length, a theoretical analysis of Q_w is not included in this paper. However, the value of Q_w is proportional to

$$\frac{a^3 b \lambda^2}{D^6\sqrt{1-\left(\frac{\lambda}{2a}\right)^2}}, \qquad (24)$$

for a given cavity diameter-to-length ratio. Fig. 8(a) illustrates cavity-waveguide dimensions as given in (24).

[6] H. A. Bethe, "Formal Theory of Waveguides of Arbitrary Cross Section," M.I.T. Rad. Lab. Rep. No. 43-26.

[7] M. Surdin, "Directive couplers in waveguides," *J. IEE*, part IIIA, vol. 93, pp. 725–736; March–May, 1956.

VI. Applications

There are many applications for the reflectionless microwave filter. The simplest filter is the band-elimination filter with two waveguide ports. Normally, a TE_{111} circularly cylindrical cavity mode, Type A coupled, would be used, since it is very small and does not require a high cavity Q. The four-port, reflectionless diplexer (Figs. 2 and 3) can be constructed in several ways. Each cavity mode and type of coupling has certain advantages and disadvantages as far as loss, available bandwidth, unwanted modes, and availability for temperature compensation are concerned. Therefore, the type of cavity to be used will depend upon the application.

A circular polarization separator may be constructed, as shown in Fig. 11, by using a single cavity, a rectangular waveguide, and a circular waveguide. At the cavity resonant frequency, energy entering port A will be coupled to arm C. At port C, the energy will be in the form of a circularly polarized TE_{11} mode. If the input was at port B, then the energy coupled to arm C would be circularly polarized in the opposite direction. Therefore, this device would be suitable as a duplexer for a fixed-frequency circularly polarized radar. The transmitter, receiver, and antenna would be connected to ports A, B, and C, respectively. The cavity would be tuned to the transmitter frequency. Transmitted energy would be coupled from arm A to arm C and would radiate from the antenna as a circularly polarized field. Target reflections would be received by the antenna and coupled from arm C to B. There would be approximately 40-db isolation between the transmitter and receiver. This system sees twice the cavity loss; therefore, a large cavity bandwidth should be used to minimize this loss.

The addition of a ferrite in the circularly polarized cavities at the point of circularly polarized magnetic field produces a reflectionless filter that is nonreciprocal and tunable. An important application of the ferrite-loaded reflectionless filter is a passive duplexer for a fixed-frequency radar (see Nelson[8]).

VII. Sample Designs and Experimental Results

Several cavities with Type A coupling have been built and tested. The tests results verify the analysis and indicate the degree of accuracy of the theoretical curves of window-coupling factor Q_w.

The reflectionless filter shown in Fig. 2 was constructed using the TE_{111} circularly cylindrical cavity mode. The internal cavity length is 0.800 inch and the diameter is 1.350 inches. Two coupling holes 0.370 inch in diameter, with a 0.020-inch wall thickness, were placed 0.212 inch from the centerline in a standard,

[8] C. E. Nelson, "Ferrite tunable microwave cavities and the introduction of a new reflectionless, tunable microwave filter," presented at the Symposium on Microwave Properties and Applications of Ferrites, Harvard Univ., Cambridge, Mass.; April 2, 1956.

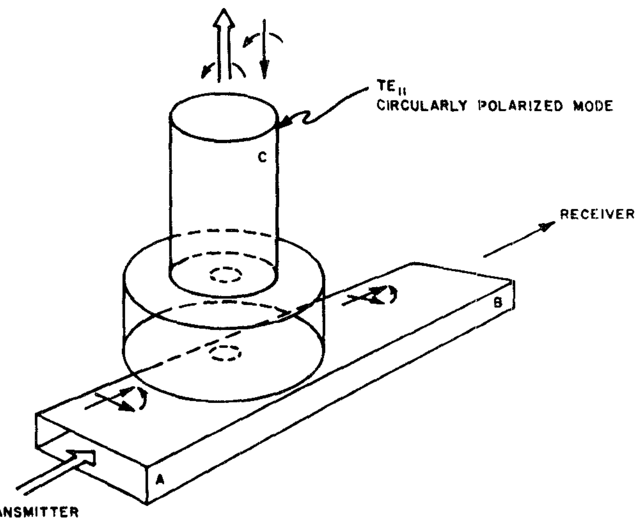

Fig. 11—Circular polarization separator.

X-band rectangular waveguide (0.400 by 0.900 inch). Five turns of an 0–80 NF metal screw, that was placed midway between the cavity end-plates, were required to reduce the resonant frequency of one cavity mode, so that both cavity modes were resonant at the same frequency (in order to produce a minimum reflection at the input waveguide). The TE_{111} cavity mode is resonant at 8755 mc, and there are no other mode resonances between 8300 and 10,000 mc. Between waveguide terminals A and C, the loss at resonance is 0.9 db and the bandwidth is 11.8 mc. The isolation between terminals A and B and between terminals A and D at resonance is 19.9 db and 27.9 db, respectively. For coupling holes of equal size, (18) may be transformed into

$$Q_w = \frac{Q_L}{\sqrt{n}}$$

$$Q_c = \frac{Q_L}{1 - \sqrt{n}}$$

$$m = (1 - \sqrt{n})^2,$$

where $n = 0.814$ for a 0.9-db loss. Using this experimental data, the calculated values of Q_L, Q_w, Q_c, and m are 742, 823, 6810, and 19.3 db, respectively. The calculated value of m (the isolation between A and B at resonance) corresponds very well with the measured value of 19.9 db. The value of Q_w is 32 per cent higher than that predicted in Fig. 10 and by (23) and (24). The cavity Q, Q_c, is 50 per cent lower than the theoretical value for solid silver cavity walls. A polished cavity with silver plated walls should have about 70 to 80 per cent of the theoretical cavity Q, because of the porosity and surface finish of the plated silver. The test cavity was slightly oxidized; this would account for the reduction in cavity Q from 70 to 50 per cent. The analysis indicates that, if the coupling hole in the main waveguide (A-B) is increased in size, then the isolation between terminals A

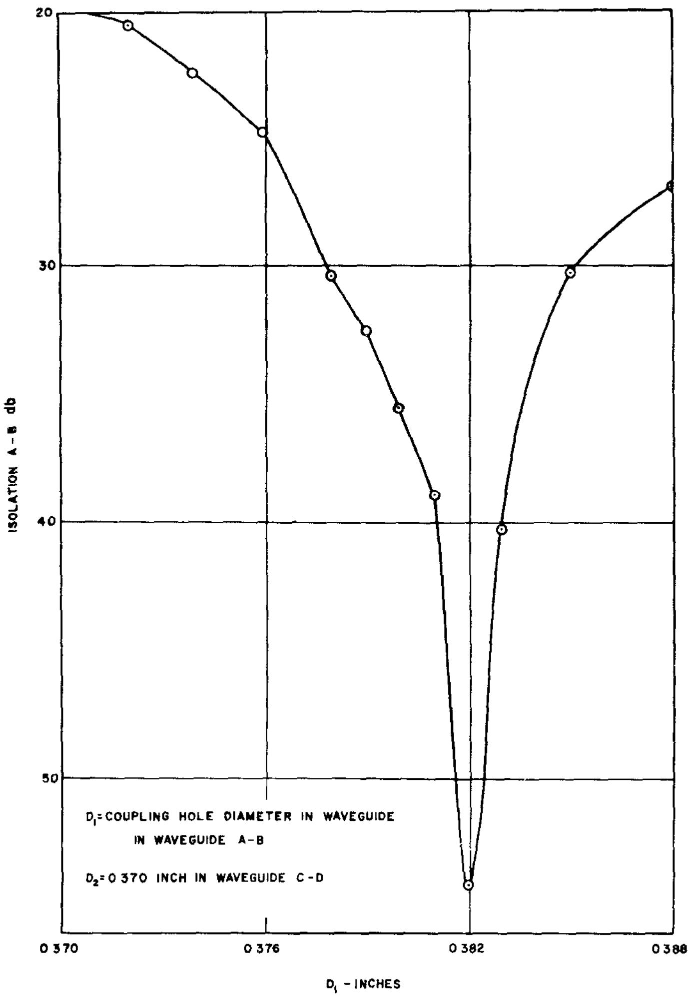

Fig. 12—Isolation between ports A and B for four-port TE_{111} mode cavity filter using type A coupling.

and B at resonance should also increase. The theoretical coupling-hole size for maximum isolation between A and B may be found from (19) and (23).

$$\frac{Q_{w2}}{Q_{w1}} = 1 + \frac{2Q_{w2}}{Q_c} = \left(\frac{D_1}{D_2}\right)^6$$

or

$$\left(\frac{D_1}{0.370}\right)^6 = 1 + \frac{2 \times 823}{6810},$$

then

$$D_1 = 0.3836 \text{ inch.}$$

If the coupling hole in waveguide A-B is 0.0136 inch larger than the hole in waveguide C-D (which is 0.370 inch in diameter), then an infinite isolation at resonance between terminals A-B should result. Fig. 12 shows the experimental measurements of isolation for increasing hole size in waveguide A-B. The maximum isolation (about 54 db) occurred with a hole 0.382 inch in diameter. Fig. 13 is a typical input vswr curve for this filter without a matching button in the waveguide. The off-resonance reactance of the coupling hole in the wave-

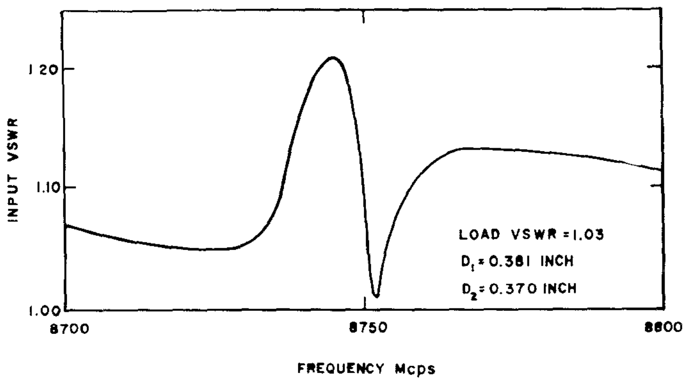

Fig. 13—Input vswr for four-port, TE_{111} mode cavity filter using type A coupling.

guide produces a vswr of about 1.10. Near resonance, the cavity impedance produces a small reflection that adds to, or subtracts from, the off-resonance reflection. This small reflection caused by the cavity impedance may be due to a slight inequality in the H_x and H_z window-coupling factors or to the finite window size.

A similar filter was constructed using the TE_{121} circular cylindrical cavity mode. The internal cavity length is 1.200 inches and the diameter is 2.590 inches. Two coupling holes, each 0.458 inch in diameter, were used. The TE_{121} cavity mode is resonant at 9107 mc, and there are other resonances at 8920, 9130, 9499, and 9869 mc (some of these are quite small). The TE_{121} cavity mode has very low cavity-wall losses; therefore, the loss at resonance should also be low. Between terminals A-C (Fig. 2), the loss at resonance is 0.5 db and the bandwidth is 6.6 mc. The isolation between terminals A and B and between terminals A and D at resonance is 27.7 db and 25.3 db, respectively. The measured value of the window-coupling factor Q_w is 39 per cent higher than the theoretical value found by using small-aperture theory. The measured value of cavity Q is 25,200. This is 70 per cent of the theoretical value. Fig. 14 illustrates the isolation between terminals A and C and between terminals A and D, for frequencies near resonance. As is typical of microwave cavities, the band-pass filter response is asymmetrical.

A reflectionless band-elimination filter was constructed that had only one waveguide and a circular cylindrical TE_{111} cavity mode [see Fig. 8(a)]. The cavity diameter and length are 1.350 and 0.800 inches, respectively. A single coupling hole, 0.360 inch in diameter, was used. Midway between the cavity end-plates, four 0–80 NF screws were equally spaced about the cavity circumference. Two of the screws were metal; the others were lossy dielectric screws. All were adjusted for approximately minimum reflection and maximum absorption at resonance. The isolation at resonance between input and output is 36 db with a 3-db bandwidth of 11 mc and a 10-db bandwidth of 3.7 mc. The loss in output power at $f_0 \pm 11$ mc is theoretically (for $m=0$)

$$\frac{(2)^2}{1+(2)^2} = -0.97 \text{ db;}$$

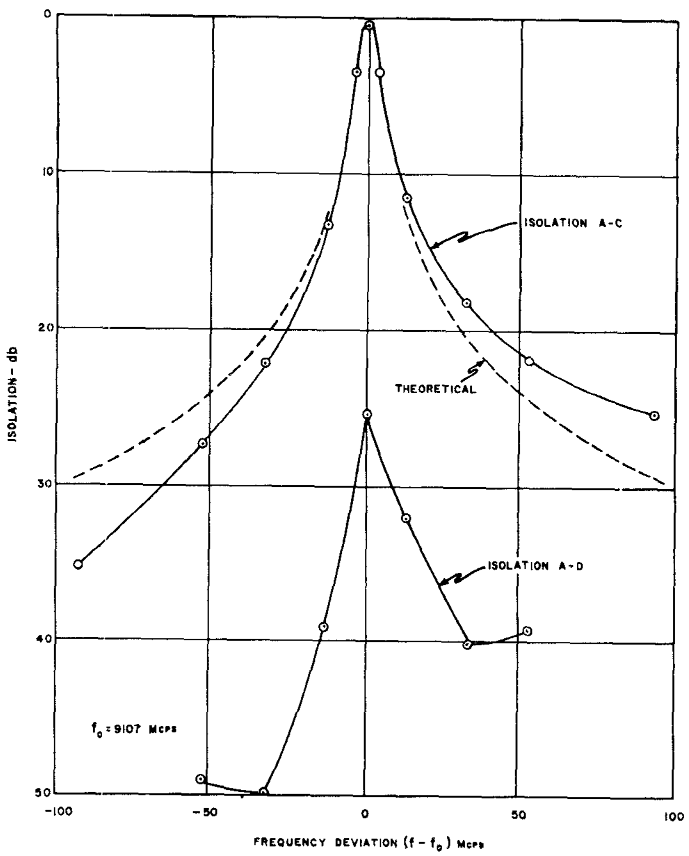

Fig. 14—Isolation between ports A and C and between A and D for four-port TE_{121} mode cavity filter using type A coupling.

the measured values above and below resonance are 1.0 and 1.1 db, respectively. The input vswr of the band-elimination filter without matching is shown in Fig. 15. The test results correspond very well with the theory.

A circular polarization separator (Fig. 11) was constructed using a TE_{111} circular cylindrical cavity mode. The cavity-wall losses were rather high, but the device functioned in the manner predicted. Without a mode-tuning screw, power into terminal A produced an elliptically polarized waveguide field at terminal C. When

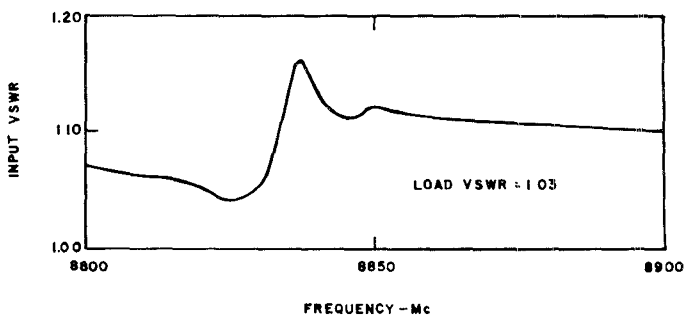

Fig. 15—Input vswr for two-port, TE_{111} mode cavity filter using type A coupling.

the tuning screw was adjusted for minimum reflection at terminal A, the vswr in the circumferential direction in arm C dropped to about 1.05. This is equivalent to a 32-db ratio in the amplitudes of the two directions of circularly polarized fields. Some experimental data on the ferrite loading of circularly polarized cavities are given by Nelson.[8]

VIII. Conclusion

Theoretical analysis proves that a single, circularly polarized cavity may be used to construct a reflectionless band-pass and band-elimination filter that is capable of coupling nearly 100 per cent of the energy from the main waveguide at resonance. The theoretical loss of the band-pass portion of this filter is identical with that in any conventional band-pass cavity filter. Among many other applications, the circularly polarized cavities may also be used in the design of diplexers and tunable nonreciprocal filters.

Experimental results verify the analysis for a degenerate cavity mode coupled to the rectangular waveguide, magnetic field components at the point of circular polarization. The input vswr of the experimental cavities is below 1.2 without matching. The amount of energy coupled from the main waveguide at resonance could be adjusted to produce an isolation greater than 50 db.